## LITERATURE CITED

1. V. P. Starr, "Factors related to the inception of tornado funnels," Riv. Ital. Geofis., 3, Nos. 1-2, 1-7 (1976).
2. A. S. Monin and R. V. Ozmidov, Ocean Turbulence [in Russian], Gidrometeoizdat, Leningrad (1981), pp. 246-298.
3. J. O. Hinze, Turbulence, McGraw-Hill, New York (1959).
4. D. I. Grinval'd, Turbulence of Channel Flows [in Russian], Gidrometeoizdat, Leningrad (1974), pp. 94-103.
5. R. Deissler and M. Perlmutter, "Analysis of the flow and energy separation in a turbulent vortex," Int. J. Heat Mass Transfer, 1, No. 1, 173-191 (1960).
6. A. S. Predvoditelev, "Molecular-kinetic foundation of the equations of hydrodynamics," Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, No. 5, 545-560 (1948).
7. C. J. Scott and D. R. Rask, "Turbulent viscosities for swirling flows in a stationary annulus," Trans. ASME, Ser. I: J. Fluids Eng., 95, No. 4, 557 (1973).
8. V. A. Bobr, I. Z. Gabdullin, V. I. Kalilets, et al., "Estimation of the swirl ratio and Kármán constant for vortices," Inzh.-Fiz. Zh., 44, No. 3, 396-401 (1983).
9. V. A. Bobr, L. Kh. Garmize, V. I. Kalilets, et al., "Modeling of atmospheric vortex formations," in: Evolutionary Problems of Energy Transfer in Inhomogeneous Media [in Russian], ITMO Akad. Nauk Bel. SSR, Minsk (1982), pp. 3-19.
10. I. Z. Gabdullin and A. A. Solov'ev, "Influence of acoustic oscillations on turbulent transfer in a vortex," in: Abstr. Third All-Union Symp. Physics of Acoustooptical and Acoustohydrodynamic Phenomena [in Russian], Nauka, Tashkent (1982), pp. 17-18.
11. C. R. Church et al., "Characteristics of tornado-like vortices as a function of swirl ratio: a laboratory investigation," J. Atm. Sci., 36, 1775-1795 (1979).
12. R. C. Gentry, "Hurricane modification," in: Weather and Climate Modification (W. N. Hess, ed.), New York (1974), p. 509.
13. J. van Miegham, Atmospheric Energetics, Clarendon, Oxford (1973).


# UNIFORM AND NONUNIFORM FLUIDIZED BED REGIMES

Yu. I. Chernyaev

UDC 532.546

By analyzing jet flows in a fluidized bed the author has obtained a relation defining the boundary between uniform and nonuniform regimes.

In accordance with the conventional terminology (see, e.g., [1-3]) we shall call a fluidized bed regime uniform if the bed can continuously expand upon an increase of velocity of the gas or liquid, due to a uniform increase of the gaps between particles of the granular material. If, on the other hand, for velocities exceeding the bed start-up velocity, the gas or liquid moves through the bed in the form of bubbles, we shall call this regime nonuniform.

Several approaches have been taken toward an explanation of the differences between these regimes, and the most important results have been obtained from the concepts of a model of two mutually permeable continua, a two-phase model combined with extremal principles, and a model based on analysis of random motions of phases.

With the two continuous media model, described in detail in [1], one can formally show the instability of a uniform fluidized state relative to small perturbations by analyzing a linearized system of equations of continuity and motion of the two phases. A considerable defect of the model is that the roots obtained for the characteristic equation, which define the rate of growth of perturbations, depend on the pressure and the dynamic viscosity of the

All-Union Scientific-Research and Structural Design Institute for System Engineering, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 47, No. 3, pp. 402-407, September, 1984. Original article submitted June 28, 1983.

 0022-0841/84/4703-1038$08.50 

solid phase, concerning the physical meaning and values of which one must make a whole series of unconvincing assumptions. Therefore, the rate of growth of the perturbations can be calculated from experimental information only for a specified fluidized system, but one cannot derive general conclusions from the analysis.

There are two other approaches which allow us to obtain criteria for establishment of a nonuniform regime, criteria which show fair agreement with experimental information. In the analysis of random motions of the phases we take nonuniformity to be the existence of correlation relationships in the motion of the particles, i.e., the appearance of aggregates (or packets) of granular material [4]. The two-phase model treats nonuniformity as the presence in the system of bubbles of the fluidizing agent of diameter (10-25)d, and the relation between the solid phase and the bubbles is determined from the condition of minimum potential energy of the particles in the field of gravity [5-8]. These criteria are quite reliable and allow us to explain the transition of a uniform regime to a nonuniform one near the onset of fluidization and the inverse transition at large velocities of the fluidizing agent. However, the final results obtained in both methods cannot be considered comprehensive, since neither takes into account, for example, the parameters of the gas distribution device which affect the bed structure. The approach proposed in the present paper allows these parameters to be taken into account.

As the physical model of the fluidized system we shall use the model proposed in [9, 10], according to which the gas passing through the bed forms channels (or jets) there with particle concentration considerably less than in the rest of the bed volume. We note that here we are speaking of channels whose length is comparable with the bed height and whose formation is not connected with the efflux of jets from the apertures of the gas distribution grid. The relation between the fluxes of fluidizing agent in the channels and in the solid phase ensures a minimum of a certain energy function, which is subtracted from the potential energy of the particles in the field of gravity and the energy due to the gas pressure gradient [9]. From a qualitative analysis of the equations of motion the authors of [9, 10] have shown that at a specific time after formation of a channel its cross sections will be closed by particles, leading to an increased pressure drop and the formation of a bubble, redistribution of the gas over the section of the equipment, formation of a new channel, a new bubble, and so on. High-speed movie pictures of a two-dimensional bed confirm that this model conforms with the actual process.

For numerical description of the collapse of a channel and the formation of a bubble we use the relations of [11, 12] which we obtained as a result of an investigation of the system of differential equations of motion of a disperse phase in the vicinity of a circular vertical bed on its Liapunov stability. It is legitimate to extend the conclusions of [11, 12] to channels of arbitrary shape, since a number of papers, e.g., [13], have shown that the field of the velocity profile is identical for the gas issuing from apertures of different configurations. The numerical values of the velocities differ here only in regard to a constant coefficient on the order of 1.

It was shown in [11, 12] that the granular material arrives in the bed from the lower part of the boundary of the gas jet described by the positive root of the characteristic equation, since the condition

$$\left(\frac{\partial^2 v}{\partial r^2}\right)_{r=b} \geqslant 7.5\,\frac{\xi\rho\nu}{a^2\rho_\text{т} d^2\Delta} \tag{1}$$

holds, in which the quantity

$$\Delta \approx d/6\,(1 + O\,(\text{Re})). \tag{2}$$

In the upper part of the flare boundary Eq. (1) does not hold, and the particles are in a state of stable equilibrium and do not come into the jet. Between the lower and upper parts there is a region of shallow equilibrium [11] in which the right side of Eq. (1) is equal to the left side. Under the influence of turbulent fluctuations of the gas velocity field this region oscillates about its mean position. When it is displaced upwards, the mass of particles, previously in a stable equilibrium position, is unstable and tends to move within the gas flare in the direction of the eigenvector for the positive root, and the projection of this vector in geometrical space lies in the horizontal plane and is directed toward the jet axis [12]. If the jet energy is not enough to accelerate the incident mass of particles, the gas flare becomes covered and a bubble forms in its upper part. According to the experimental investigations of [13], this picture is observed for a ratio of flare height to bed height $x_f/H < 0.6$. For $x_f/H > 0.6$ the mass of incident particles is accelerated quickly and removed from the jet volume, and therefore a bubble does not form. We note that these laws are valid for the so-called "active" jets for which air is supplied from an individual blower, and the gas flare parameters are determined by the compressor operation. We are considering jets formed as a result of particles slipping past the fluidizing agent, and therefore the arrival of an added mass of granular material into such a jet is always accompanied by collapse of the gas flare and formation of a bubble [9, 10]. This is evidently due to the fact that less energy is expended in forming a new channel than on accelerating the particles arriving in the old channel. Of course, the flare will be stable if the granular material over its entire boundary is in stable equilibrium, and for this the requirement is that Eq. (1) does not hold at a distance of more than d/2 from the gas distribution grid. This will ensure discharge into the bed of jets forming without circulation of particles, without collapse, and without bubble formation, i.e., uniform fluidization in the traditional sense.

To obtain quantitative estimates one must write the second derivative of the axial flow velocity with radius, in the initial section of the jet in explicit form, on the left side of Eq. (1). It is recommended in [13] that the experimental values of the gas velocity in the initial section be approximated by the relation

$$v = v_c - (v_c - v_0)\left[1 - \left(\frac{b-r}{b-y}\right)^{1.5}\right]^3. \tag{3}$$

However, near the jet boundary ($r \sim b$) such a velocity profile cannot be realized, since to satisfy

$$\left(\frac{\partial^2 v}{\partial r^2}\right)_{r=b} = \infty \text{ and } \left(\frac{\partial v}{\partial r}\right)_{r=b} = 0$$

simultaneously contradicts the Navier–Stokes equation. To avoid this contradiction we shall use the universal Schlichting profile

$$v = v_0 + (v_c - v_0)\left[1 - \left(\frac{r-y}{b-y}\right)^{1.5}\right]^2, \tag{4}$$

which is physically realizable, can be obtained analytically from Prandtl turbulence theory, and for $r/b \geq 0.8$ describes the experimental velocity field of the gas [13] at least as well as Eq. (3). From Eq. (4) we find

$$\left(\frac{\partial^2 v}{\partial r^2}\right)_{r=b} = 4.5\frac{v_c - v_0}{(b-y)^2}. \tag{5}$$

At distances on the order of d/2 from the gas distribution grid we have $b - y \sim d$. Then from Eqs. (1) and (5) we obtain the condition for discharge of a jet into a fluidized bed without collapse and bubble formation

$$v_c - v_0 < k_1 \frac{\xi \rho \nu}{a^2 \rho_T \Delta}, \tag{6}$$

where $k_1 \sim 1$.

Below we shall investigate a bed for which Re >> 1 for the operating velocity of the fluidizing agent, and therefore Eq. (2) is simplified:

$$\Delta \sim \frac{\nu}{6 a v_0}. \tag{7}$$

The coefficient $\xi$ for a particle at the jet boundary is determined from the condition that the forces of gravity, buoyancy, and drag are in equilibrium, and the Magnus force and moment are zero [11]:

TABLE 1. Example of Fluidized Regimes Calculated from Eq. (10)

| No. | Fluidizing agent | Pressure, $MN/m^2$ | $d \cdot 10^3$, m | $\rho_T$, $kg/m^3$ | $v_0 \cdot 10^2$, m/sec | Fr | Right side of Eq. (10) | Fluidization regime from Eq. (10) | Experimental confirmation |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Air | 0,1 | 1,0 | 1000 | 30,0 | 9,18 | 2 | Nonuniform | [1—3] |
| 2 | Air | 0,1 | 5,0 | 10 | 8,0 | 0,13 | 5 | Uniform | [3, 16] |
| 3 | Air | 10,0 | 1,0 | 1000 | 5,0 | 0,26 | 3 | » | [1, 17] |
| 4 | Water | 0,1 | 1,0 | 1500 | 0,4 | 0,0016 | 0,1 | » | [1—3] |
| 5 | Water | 0,1 | 0,3 | 10000 | 0,6 | 0,012 | 0,1 | » | [1, 3, 18] |
| 6 | Water | 0,1 | 0,3 | 10000 | 9,0 | 2,76 | 2 | Nonuniform | [1, 3, 17, 18] |

$$\frac{\pi d^3}{6} g(\rho_T - \rho) = \frac{\pi d^2}{4} \xi \frac{\rho v_0^2}{2},$$

whence we find

$$\xi = \frac{4}{3} \frac{\rho_T - \rho}{\rho \, \mathrm{Fr}}. \tag{8}$$

In the physical model of the fluidized bed considered the particle concentration in the jet is small, and therefore the drag of the channel can be taken as equal to the drag of its walls, and calculated from the formulas for roughened tubes. As follows from [14] the drag coefficient of such a tube is at least an order of magnitude less than the coefficient $\xi_g$ of the gas distribution grid, if the bed height does not exceed 2500d, which holds in practice, as a rule. Therefore, one can calculate the pressure drop in a jet concentrated at the grid and equal to the pressure drop of the fluidizing agent between the zone below the grid and the volume above the bed:

$$\xi_g \frac{\rho v_c^2}{2} = \xi_g \frac{\rho v_0^2}{2} + (\rho_T - \rho) g h_0 (1 - \varepsilon_0). \tag{9}$$

The drag coefficient $\xi_g$ is taken to be the same for parts of the grid under the channel and under the solid phase of the bed, since the flow regime in the grid apertures is turbulent [14].

Substituting Eqs. (8) and (9) into Eq. (6), we obtain the condition for uniform fluidization

$$\mathrm{Fr} < k_2 \frac{v_0}{\sqrt{v_0^2 + \frac{2gh_0(1-\varepsilon_0)(\rho_T - \rho)}{\rho \xi_g}} - v_0} \frac{\rho_T - \rho}{a \rho_T}, \tag{10}$$

where $k_2 \sim 10$. This last equation means that if the Fr number is less than a certain value, depending on the physical properties of the particles, the fluidizing agent, the distribution grid, and the operating velocity, then fluidization occurs without formation of bubbles. The validity of the analogous condition (Fr < 1) was shown semiempirically in [15]. The real value of Eq. (10) is to relate the critical value of Fr to the initial bed height and the grid drag, in contrast with conditions for transition to the nonuniform regime in [4, 8, 15].

Table 1 gives examples of calculations using Eq. (10) for various regimes. In all the examples $h_0 = 0.3$ m, the temperature of the fluidizing agent is 20°C, and $\xi_g$ is calculated [14] for a perforated grid with aperture diameter of 1 mm and an actual cross section of 3% for fluidization with air and 10% for fluidization with water.

The calculations performed show that Eq. (10) gives good agreement with the known experimental data.

It follows from analysis of Eq. (10) that low beds and beds above grids with high drag (e.g., above porous slabs) will achieve more uniform conditions. In addition, if a bed is uniform at velocities close to the start of fluidization, and if

$$v_0^2 \ll \frac{2gh_0(1-\varepsilon_0)(\rho_T-\rho)}{\rho\xi_g},$$

then with increase of the operating velocity the right side of Eq. (10) increases in proportion with $v_0$, and the Fr number increases as $v_0^2$. Therefore, when a certain value of $v_0$ is reached, condition (10) breaks down and the fluidization regime becomes nonuniform, as is observed in practice [1-3] (see lines 5 and 6 of Table 1). On the other hand, if the bed is uniform when the condition

$$v_0^2 \gg \frac{2gh_0(1-\varepsilon_0)(\rho_T-\rho)}{\rho\xi_g} \tag{11}$$

holds, which occurs in low beds operating with trickling liquids and high-pressure gases, then Eq. (10) takes the form

$$1 < k_2 \frac{d\xi_g\rho}{a(1-\varepsilon_0)h_0\rho_T} \tag{12}$$

and generally does not contain the operating velocity. A fluidized system satisfying Eqs. (11) and (12) will remain uniform at arbitrarily large velocities of the fluidizing agent. Apropos of this it was shown in [3] that "a deliberate attempt to obtain a nonuniform bed of ion-exchange resin with particle dimension 0.4-0.5 mm in narrow long tubes (more than a meter) did not succeed; with increase of water velocity the bed expanded, but the uniformity of particle distribution in the liquid was not disturbed." It is easy to verify, using Eq. (12), that to obtain nonuniformity in this case one must take an initial bed height of not less than 3-5 m.

Industry makes widespread use of fluidization by air of solid particles of dimension 0.5-5 mm and density about 1000 kg/m$^3$. From Eq. (10) one can estimate the initial bed height for which the fluidization regime will be uniform. For example, for d = 1 mm, $\rho_T$ = 1500 kg/m$^3$, $v_0$ = 2 m/sec and $\xi_g$ = 500-3000 [14] one must maintain $h_0$ < 10 mm, which is hardly of practical interest.

## NOTATION

v, axial flow velocity in the channel; $v_c$, initial efflux velocity in the channel; $v_0$, operating velocity of the fluidizing agent with the equipment empty; d, $\rho_T$, $\xi$, diameter, density, and aerodynamic drag coefficient of a particle [11]; $\rho$, $\nu$, density and kinematic viscosity of the fluidizing agent; $\varepsilon_0$, $h_0$, porosity (fraction of voids) and height of the bed at rest; b, y, r, jet radius, radius of the potential core of flow in the jet, and distance from the jet axis to the center of the particle; $\xi_g$, drag coefficient of the gas distribution grid; $a$, coefficient accounting for flow constraint [11]; Fr = $v_0^2/gd$, Froude number; Re = $v_0 d/\nu$, Reynolds number; $k_1$ and $k_2$, numerical coefficients in Eqs. (6) and (10).

## LITERATURE CITED


1. J. F. Davidson and D. Harrison (eds.), Fluidization, Academic Press (1971).
2. O. M. Todes and O. S. Tsitovich, Equipment with a Fluidized Granular Bed [Russian translation], Khimiya, Leningrad (1981).
3. N. I. Gel'perin, V. G. Ainshtein, and V. B. Kvasha, Basic Fluidization Technology [in Russian], Khimiya, Moscow (1967).
4. Yu. A. Buevich and V. G. Markov, "The structure of equilibrium pseudoturbulence in gas suspensions in conditions of local nonuniformity," Zh. Prikl. Mat. Teor. Fiz., No. 5, 49 (1969).
5. I. A. Vakhrushev and V. A. Basov, "Nonuniform fluidization," Khim. Promyshl., No. 6, 401-405 (1968).
6. K. Doichev, "Energetic aspects of the transition between aggregative and particulate fluidization," Chem. Eng. Sci., 29, No. 5, 1205-1210 (1974).
7. K. Doichev and G. Boichev, "Investigation of the aggregative fluidized bed voidage," Powder Technology, 17, 91-94 (1977).
8. Yu. A. Buevich and A. N. Deryabin, "Establishment of a nonuniform fluidization regime with a uniform distribution of fluidizing agent flow," Inzh.-Fiz. Zh., 36, No. 3, 416-425 (1979).

9. N. I. Gel'perin, B. M. Grakhovskii, and V. M. Dement'ev, "The dynamics of gas bubble formation in a nonuniform fluidized bed," Teor. Osn. Khim. Tekhnol., 3, No. 6, 874 (1969).
10. B. M. Grakhovskii, "Investigation of certain hydrodynamic laws of the nonuniform fluidized bed," Author's Abstract of Candidate's Dissertation, Technical Sciences, Moscow (1969).
11. N. N. Prokhorenko and Yu. I. Chernyaev, "Discharge of a turbulent jet into a monodisperse fluidized bed," Inzh.-Fiz. Zh., 43, No. 6, 913-918 (1982).
12. Yu. I. Chernyaev, "Discharge of jets into a fluidized bed," in: Heat and Mass Transfer in Disperse Systems [in Russian], ITMO im. A. V. Lykova, Minsk (1982), pp. 60-65.
13. Yu. A. Buevich and G. A. Minaev, "Mechanics of jet flows in a granular bed," in: Reports of the International School on Transfer Processes in Fixed and Fluidized Granular Beds [in Russian], ITMO im. A. V. Lykova, Minsk (1977), pp. 3-48.
14. I. E. Idel'chik, Handbook on Hydraulic Drag [in Russian], Mashinostroenie, Moscow (1975).
15. R. H. Wilhelm and M. Kwauk, "Fluidization of solid particles," Chem. Eng. Prog., 44, No. 3, 201-218 (1948).
16. D. F. Otmer, Processes in the Fluidized Bed [Russian translation], K. P. Lavrovskii (ed.), Gostoptekhizdat, Moscow (1958).
17. H. C. Simpson and B. W. Rodger, "The fluidization of light solids by gases under pressure and heavy solids by water," Chem. Eng. Sci., 16, No. 3, 153-180 (1961).
18. L. Massimilla, G. Volpicelli, and F. A. Zenz, "Flow of fluid-particle suspensions from liquid-fluidized beds," Ind. Eng. Chem. Fund., 2, No. 3, 194-199 (1963).

# THERMODYNAMIC PROPERTIES OF n-BUTANE

E. T. Vas'kov

UDC 536.7

Equations are derived and used to calculate a table of thermodynamic properties of n-butane in the saturated state.

In calculating chemical engineering processes and refrigeration equipment cycles in which n-butane is used as the raw material or cooling agent, reliable data on the thermodynamic properties of that material are essential.

Below we will present self-consistent equations for pressure, density, specific heat, heat of evaporation, enthalpy, and entropy, which were obtained by critical evaluation of experimental data. The equations were used to calculate a table of thermodynamic properties for n-butane in the saturated state, suitable for practical use.

Saturated vapor pressure was calculated with the equation

$$\lg p = A + BT^{-1} + CT + DT^2 + ET^3 + KT^4, \tag{1}$$

which to an accuracy of $\pm 0.2\%$ describes the experimental data of [1-4] on saturated vapor pressure of n-butane over the temperature range from the triple point to the critical point. In Eq. (1) pressure is expressed in MPa and temperature in °K.

The constants appearing in Eq. (1) were determined by the method of least squares: A = 13.9878; B = −3597.20; C = $-8.05251 \cdot 10^{-3}$; D = $-3.05786 \cdot 10^{-5}$; E = $8 \cdot 13812 \cdot 10^{-8}$; K = $-4.57782 \cdot 10^{-11}$.

Density of the boiling (saturated) liquid has been measured over the following ranges: [5] (273-32°K); [6] (293-306°K); [7] (294-394°K); [2] (325-422°K); [8] (227-333°K); [9] (135-275°K); [10] (93-173°K); [11] (135-300°K); [12] (283-368°K); [13] (at 293 and 298°K); [14] (311-411°K); [15] (at 288.65 and 327.55°K). The data of [9-12] are the most reliable and precise.

Leningrad Mechanical Engineering Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 47, No. 3, pp. 407-410, September, 1984. Original article submitted April 4, 1983.

0022-0841/84/4703-1043$08.50  